Capital Mobility, Quasi-Rents, and the Competitive Self-Organization of Distributions of Profitability

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Abstract

This paper considers patterns of organization in distributions of the rate of return on capital (RoC) realized by individual enterprises. It shows that large-sample cross sections of RoC across several European economies are persistently well described by the same functional form: Sharply peaked distributions with stretched-exponential tails. To account for this observation, the paper develops a systemic model of the competitive regulation of profitability by the pursuit of arbitrage profits latent in any heterogeneity across values of RoC. Under the model, the observed distributional forms embody a simple, emergent result of capital-market competition: The competitive tradeoff between aggregate pecuniary returns and costs accruing to those reallocating capital according to profitability differentials. They also reflect the presence of dynamic entrepreneurial and monopolistic quasi-rents. The paper's discussion defines a series of new, observable macroscopic measures of competitive performance in decentralized market economies. It also points to the aptness of understanding prices as parts of structures of generalized Marx-Sraffa "prices of production," predicated on the characteristics of capital-market statistical equilibria; to a general theoretical approach to the regulation of certain economic quantities by arbitrage; and to the role the costs of informational gains play in shaping observable outcomes in the operation of certain types of goal-seeking, self-organizing systems.

Keywords: Information Theory, Distributions of Profitability, Self-Organization, (c, d) entropies, Observational Economics

1 Introduction

The quest for profitability is the defining organizing principle of decentralized capitalist economies. Enterprises organize all aspects of the production and sale of goods and services so as to maximize

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the rate of return on assets represented by their profits. Realized measures of profitability reflect the functioning of markets for labor, capital, and for the inputs employed and outputs produced by enterprises. Together with expectations about future profitability, those measures also inform the allocation of capital, shaping the extent to which different undertakings are supported by investors. As a result, distributions of profitability should contain important formal clues about the nature and dynamic evolution of complex economic systems.

A number of contributions have explored different aspects of this expectation. Some drew on theoretical insights to speculate about the shape of those distributions.[1, 2] Others sought to investigate the empirical purchase of these and other Classical contentions about the effects of competition on observable moments of distributions of profitability.[3, 4, 5]

A more recent set of contributions opened a different, *observational* line of inquiry, centering on efforts to identify the empirical form taken by distributions of profitability as a first step informing the development of theories of competition in capitalist economies.[6, 7] Drawing on data from U.S. non-financial corporations, those studies found evidence that the annual, cross-sectional distributions of firm-level profitability are consistently well-approximated by the power-exponential forms given by asymmetric Subbotin distributions. This work led to two initial theoretical contributions—broadly grounded on the Classical contention that the mobility of capital tends to equate realized measures of profitability at an emergent *general rate of profit*—that account for those distributions as statistical equilibria defined either by specific micro-kinetic, drift-diffusion processes,[8] or by the "bounded rationality" of "inattentive" or informationally constrained individual arbitrageurs in capital markets.[9]

Drawing on Information Theory and on critical applications of theoretical insights from Classical and Austrian political economy, the present paper makes a series of contributions concerning the empirical shape and economic significance of observed distributions of profitability.

Empirically, the paper offers a full account of recent, summarily reported results that confirmed, broadened, and extended the finding of persistent formal regularities in distributions of measures profitability.[10] Drawing on qualitatively larger $\left(\sim 10^4 - 10^5\right)$, nationally representative samples of private and publicly traded companies across fifteen European economies, it establishes that the tails of country-level cross-sectional distributions of realized firm returns on capital (RoC) are persistently well-described by the generalized power-exponential forms given by *stretched exponential* distributions.

The persistence and ubiquity of these distributional forms across individual measures of profitability that are shaped by complex, dynamic patterns of interaction among enterprises and other economic actors are remarkable. They also enable the pursuit of a systemic methodological approach to inquiry into the competitive regulation of realized measures of RoC.[11] Instead of offering micro-level accounts based on strong specifications of *unobservable* characteristics of individual agents, markets, or evolutions, the paper considers the persistently observed macroscopic distributional patterns as the emergent outcome of complex micro-level competitive dynamics that economic theory can and ought to explain directly.[12, 13, 14, 15]

The paper provides a *systemic*, reduced-form explanation of the observed patterns of organization based on a distinctive formalization of the Classical account of the competitive regulation of realized measures of profitability by the mobility of capital,[16, 17, 18] and an Austrian understanding of the emergent properties of the aggregate properties of the outcomes of competitive market processes.[19, 20] It does so with the aid of an *informational accounting* of the dynamic evolution of a decentralized, competitive economy:[21, 22] a formal characterization of the effects competitive interactions and broader economic changes have on the informational characteristics of distributions of RoC. This yields a generalized Classical theoretical model of competitive self-organization over a domain that is central to the functioning of complex economic systems. Under the model, the distributional forms we observe express rather simple economic and combinatorial facts.

In a decentralized market economy driven by the push for individual pecuniary gains, organization or entropy reductions in distributions of realized measures of profitability are the unintended result of a specific type of competitive intervention: The pursuit of what this paper terms *arbitrage* profits, which may be generated by effectively moving capital value and broader competitive efforts from low- to high-profitability undertakings. By generally contributing to reductions in returns on high-yield undertakings and to increases in low-yield ones, this movement tends to push all rates of return toward an emergent, general rate of return r_g that offers the benchmark measure of the *opportunity cost of capital* against which all rates of return are measured. By concentrating rates of return around r_g, the pursuit of arbitrage profits latent in any heterogeneity across rates of return in the system results in gross informational gains in those distributions. In contrast, all other competitive or entrepreneurial actions, and broader economic developments or changes, are overwhelmingly likely to generate gross entropy increases in distributions of RoC.

It is thus possible to understand the patterns of organization in cross-sections of RoC as the spontaneous, systemic outcomes of large numbers of unobservable individual efforts to realize arbitrage profits within broader, complex competitive processes. The paper identifies two emergent, economic characteristics of those competitive outcomes that can account for the distributions we observe.

First, the manner in which the general or opportunity rate r_g is endogenously formed on the basis of contemporaneous individual measures of RoC reflects the presence of dynamic entrepreneurial and monopolistic quasi-rents. Dynamic quasi-rents or atypically high measures of profitability can ensure that the competitive regulation of measures of profitability results in the persistent right skews observed in distributions of RoC. Second, competitive capital-market processes and price signals ensure that even though we cannot observe the micro-level details of the pursuit of arbitrage profits, we can reasonably conclude that its outcomes reflect an aggregate economic tradeoff between gross returns and gross costs accruing to those seeking those profits. Significantly, this tradeoff is shaped by the fact that over any given time period, the aggregate costs of the actions of arbitrageurs are overwhelmingly likely to be increasing on the gross measure of entropy reduction the actions of arbitrageurs effect on distributions of RoC. Under the model developed by the paper, those distributions simply embody the pecuniary pricing of informational measures of organization emerging from capital-market competition.

In addition to offering an economic account of the observed distributions of RoC, the paper's model motivates a series of significant additional points. It defines a number of macroscopic measures of competitive performance. It also supports the understanding of those distributions as *statistical equilibria.* This in turn enables a characterization of competitive prices as parts of generalized systems of Classical "prices of production" conditioned by wage structures, productive techniques, and statistical equilibria in capital markets. It also offers a general approach to the dynamic regulation of certain economic quantities by arbitrage, and to the functioning of a class of goal-driven, self-organizing systems that are often relevant to social and economic inquiry.

The paper proceeds as follows. Section 2 presents the evidence we have considered. Section 3 offers a discussion on the methodological implications of macroscopic stability in distributions of profitability, the explanatory burdens it places on theories of competition, and on recent attempts to offer a theoretical account for the central form of regulation of profitability around its modal value. Section 4 offers the conceptual discussion of the economic contentions at the heart of this paper. Section 5 offers a formal statement and solution of the model implied by that discussion. Section 6 concludes with a discussion of lines of further work suggested by the paper.

2 The Distributions, Stretched-Exponentials, and Statistical Equilibria

This section offers a full and detailed account of results and conclusions first presented in summary form in a recent letter.[23] It reports on the observed series of end-of-year distributions of RoC across a number of European economies, discusses the remarkably good performance of stretched-exponential models, and motivates the use of those functions as *statistical equilibrium* models of the result of complex competitive interactions in markets for goods, labor, and capital. The mathematical characteristics of such equilibria define the formal burden on any successful theorization of those processes.

2.1 The Data and Distributional Fits

We used data from the Amadeus Company Information Database,[24] which currently contains data on more than 20 million individual European enterprises. The database contains samples that are representative of the structure of each economy, including firms of all sizes. Approximately 99 percent of them are private, non-corporate organizations.

We constructed national, annual frequency histograms for each year between 2007 and 2015 for the ratio r_i of companies' "Earnings Before Interest, and Tax" (EBIT) to their "Total Fixed Assets." We considered fifteen economies with more than 10,000 observations per year during the period in question. This resulted in a set of fifteen European economies, including Germany, France, the UK, Italy, and Spain. While in the earlier, summary letter we considered non-financial companies with RoC between -200 and 200, here we report on a much wider range of those companies—those with RoC between -1000 and 1000 percent. This is a very wide, economically relevant range beyond which observed frequencies are typically three orders of magnitude smaller than those over central values of r_i. This resulted in fifteen national series, each with several end-of-year cross sections. Those cross sections are densely populated, with an average of 94,758 enterprises in each. Italy, Spain, Portugal, and France have the highest average annual samples, with 281,187, 2427,253, 162,409, and 121,354 enterprises respectively. Estonia and Slovenia had the smallest, with 16,496 and 16,045 respectively.

From this data we generated cross-sectional histograms for the measure of RoC centred around the mode r_g of each distribution, $x_i = r_i - r_g$, using a coarse graining into bins with RoC values x_k. Those histograms show a strikingly consistent pattern of organization, as shown in the plots for x_k, as shown in Figure 1 for Germany, France, Italy, and Spain (see Appendices for all plots).

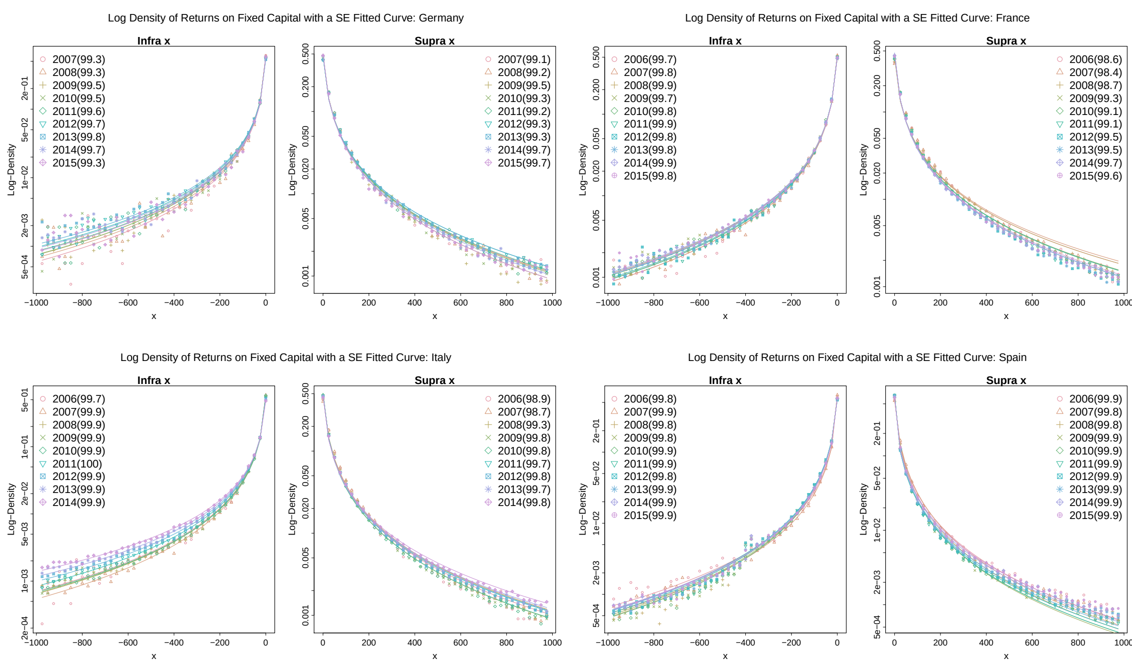

Figure 1: Observed Log-Frequencies and Minimum KL-Divergence Fits for Stretched-Exponential Model, Germany, France, Italy, and Spain, with Informational Indistinguishability Index insets, 2007-2015.

In our earlier, summary contribution, we established remarkably good piecewise fits for stretched-exponential models for the distributions of $x_k = r_k - r_g$,

$$f_k = \exp\left[1 - (\lambda_n x_k + \mu_n)^{\frac{1}{d_n}}\right], \quad x_k < 0 \quad ; \quad f_k = \exp\left[1 - (\lambda_s x_k + \mu_s)^{\frac{1}{d_s}}\right], \quad x_k > 0 \tag{1}$$

for $d, \lambda_s, \mu_s, \mu_n > 0$ and $\lambda_n < 0$. Each of these tails is subject to a normalization constraint given, in a piecewise description of the entire distribution across all x_k with a as the proportional mass of the right tail, by,

$$-\lambda_n = \frac{e\, d_n}{1-a}\Gamma\left(d_n, \mu_n^{\frac{1}{d_n}}\right) \quad ; \quad \lambda_s = \frac{e\, d_s}{a}\Gamma\left(d_s, \mu_s^{\frac{1}{d_s}}\right) \tag{2}$$

These implicitly define $\mu_n = \mu_n(d_n, \lambda_n)$ and $\mu_s = \mu_s(d_s, \lambda_s)$ over the parameter ranges under consideration.

Using the minimization of the Kullback-Leibler divergence as a criterion for parameter selection, models $\mathbf{f}(d, \lambda)$ based on 1 and 2 offer very good fits for the observed histograms $\mathbf{h}$, as measured by the informational indistinguishability index $\mathrm{ID}_{\mathbf{h}||\mathbf{f}(d,\lambda)}$ between models and observations.[25, 26]. As reported in Table 1, the estimated stretched-exponential models capture a very high proportion of the informational content of the observed histograms. The all-country-year average $\mathrm{ID}_{\mathbf{h}||\mathbf{f}(d,\lambda)}$ stands at 99.53 for all left tails and at 99.45 for all right tails. Significantly, the measures are even better for the two large-sample, high-income economies in the dataset, with average values for informational indistinguishability for left and right tails of, respectively, 99.91 and 99.49 for Italy and 99.87 and 99.89 for Spain.

The stretched-exponential models also provide a modestly better two-parameter, power-exponential description of the observed distributional tails than estimated Subbotin distributions. This may also be seen summarily in Table 1, which reports average values for two comparative measures of goodness of fit between the two models, the difference between their ID indices and their likelihood ratio. Full reports of all country-year goodness of fit measures, estimated parameters and their standard errors are provided in the Appendix.

The evolution of estimated parameter values are depicted in Figures 2 and 3 for the five largest, advanced economies we observed, with one-standard-deviation ranges for their value obtained with bootstrapping. The full set of estimated parameter values and their standard errors is provided in the Appendix.

	ID for SE fits		Diff with ID for GN fits		Likelihood Ratio SE-GN		Sample Size	% of Total
Country	left tail	right tail	left tail	right tail	left tail	right tail		
Bulgaria	99.25	99.04	0.27	0.34	1.003	1.004	84629	93%
Czech Republic	99.78	99.66	0.28	0.30	1.003	1.003	54122	94%
Estonia	99.37	99.64	0.24	0.17	1.003	1.002	16496	94%
Finland	99.14	98.66	0.12	-0.22	1.001	0.998	38429	94%
France	99.81	99.14	0.57	-0.15	1.006	0.999	121354	95%
Germany	99.55	99.37	0.52	0.36	1.005	1.004	37145	95%
Hungary	99.69	99.62	0.21	0.36	1.002	1.004	64991	91%
Italy	99.91	99.49	0.45	0.16	1.005	1.002	281187	96%
Portugal	99.49	99.68	0.23	0.51	1.002	1.005	162409	92%
Romania	98.69	99.36	-0.24	0.02	0.997	1.000	83425	87%
Slovakia	99.63	99.58	0.12	0.12	1.001	1.001	42986	95%
Slovenia	99.51	99.79	0.38	0.54	1.004	1.005	16045	94%
Spain	99.87	99.89	0.22	0.54	1.002	1.005	247253	97%
Sweden	99.70	99.08	0.23	-0.22	1.002	0.998	118863	89%
United Kingdom	99.63	99.77	0.14	0.53	1.001	1.005	52041	87%
Average	99.53	99.45	0.25	0.22	1.002	1.002	94758	93%

Table 1: Absolute and Comparative Average Goodness of Fit Indices – Informational Indistinguishability (ID) for Stretched-Exponential (SE) Fits, Differences between ID for SE fits and Generalized Normal (GN) or Subbotin fits, Likelihood Ratio between SE and GN fits, and Average Absolute and Relative Sample Size.

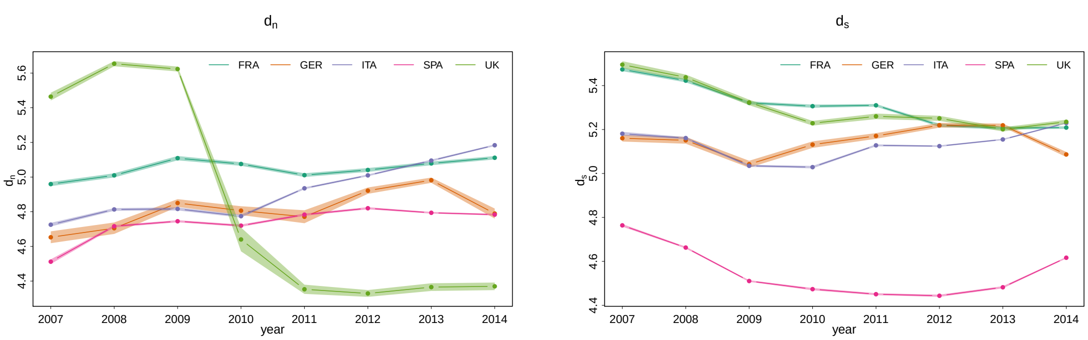

Figure 2: Estimated values for d_n and d_s, Germany, France, UK, Italy, and Spain, 2007-2015.

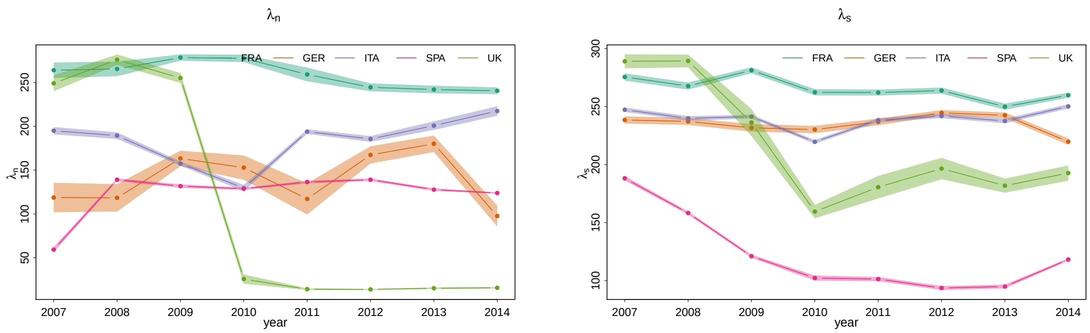

Figure 3: Estimated values for λ_n and λ_s, Germany, France, UK, Italy, and Spain, 2007-2015.

Particularly since the crises and instability of 2007-2010, the estimates for all four parameters are fairly stable for almost all countries, relative to observed cross-country variations. Among the major economies, the UK is the most notable exception, exhibiting a clear change in the range of values taken by all four parameters between the crisis and post-crisis period. Notably, the estimated values for d_n and d_s for all economies lie well within 4 and 5.8, a fairly narrow range compared to the difference between estimated values for those variables and 1. This strongly suggests d_i and d_k are significantly different from 1, a point to which we return below.

2.2 Statistical Equilibria at Stretched Exponentials

As shown in a number of recent contributions,[27, 8, 28, 11, 29, 15, 30] many important economic quantities x_i have been observed to take individual values supporting cross-sectional macroscopic states—defined over a coarse graining of values of x_i into s bins at values x_k—that are persistently and ubiquitously well described by well-known functional forms $\mathbf{f}$. In line with common practices in observational inquiry, the theoretical task posed by this finding is the development of formal accounts of the processes shaping values of x_i that are mathematically equivalent to those functional forms. This is different from the typical approach taken by contemporary economic inquiry, which starts from characterizations of expected relationships on the basis of "first principles" taken to define economic behavior, and then proceeds to "test" those with relationships with observed data.

A persistently observed functional form offers a good *statistical equilibrium* model of the macroscopic, systemic, or reduced-form outcomes of the complex and typically unobservable micro-level processes shaping individual states across the N members of the system. This is because such observation strongly suggests that $\mathbf{f}$ is the most common macroscopic state supported by the micro-level configurations of individual states across all possible values of x_k generated by the functioning of the system. In many cases, this conclusion can yield formal insights into the macroscopic functioning of the system in question.

Formally, let Ξ be the *phase space* of the system: the set whose elements correspond to each and every possible micro-level configuration for the system. Each element in $\xi \in \Xi$ supports a macroscopic state $\mathbf{f}$. Let Φ be the set of all macroscopic states $\mathbf{f}$ supported by all elements of Ξ. Naturally, each $\mathbf{f} \in \Phi$ is supported by a multiplicity of micro-level configurations. This multiplicity can be understood geometrically, as a measure Ω of the volume in the space Ξ occupied by micro-configurations resulting in $\mathbf{f}$. The entropy $S_{\mathbf{f}}$ of a macroscopic state $\mathbf{f} \in \Phi$ is an informational measure of Ω. It is also a measure of the heterogeneity across individual values of x_i in the system.

If we have a knowledge set W suggesting a particular formal characterization of the set Φ, we should rationally expect to observe the system in states at or very "close" to the distribution $\mathbf{f}^*$ achieving the maximum possible measure of entropy in that set.[31] For systems where $N >> s$, the statistical dominance of the maximum-entropy state over all other $\mathbf{f}$ is typically overwhelming. For those systems, spontaneous reductions in entropy over time are overwhelmingly unlikely. In fact, observation of macroscopic configurations or behavior at odds with a state $\mathbf{f}^*$ strongly suggests that our knowledge set W is either incorrect or incomplete.

Often, the functional forms offering a statistical-equilibrium model are known to maximize entropy over sets Φ that can be characterized through sets of macroscopic constraints or statistical statements. In those cases we can conclude that those constraints or statements are formal, macroscopic or systemic expressions of the micro-level functioning of the system in question. The identification of those constraints can help the development of theoretical accounts of the economic processes generating the aggregate patterns we observe.

For many systems, the correct description of the phase-space volume supporting a state $\mathbf{f} = \{f_1, ..., f_s\}$ bins is given by the multinomial factor, which measures the number of ways in which the system's N members can be distributed across bins with occupancies Nf_k,

$$\Omega_{\mathbf{f}} = \frac{N!}{\prod_k (Nf_k)!} \tag{3}$$

In these cases the Shanon entropy, $S = \log \Omega^{\frac{1}{N}}$, provides the correct informational measure of the phase-space volume supporting any given macroscopic state—the average number of bits, dits, or nats (depending on the base of the logarithm) needed to enumerate all micro-level configurations supporting that macroscopic state.

For many other systems, multinomial coefficients do not offer the correct measure of phase-space volumes. As shown in an important recent set of contributions from mathematical physics, complex systems characerized by strong, dynamic interactions between their individual members are not generally multinomial.[32, 33] Interactions render individual states interdependent, which in turn shape how phase-state volumes vary according to the number of interacting members in the system. In those cases, generalized (c, d) entropy functionals offer the correct measure of phase-space volume. The Shannon entropy is a special case of those functionals, with $d = c = 1$.

The stretched-exponential tails in 1 and 2 maximize generalized $(1, d)$ entropy functionals among all distributions $\mathbf{f}$ subject to a first-moment contraint $\langle x\rangle_{\mathbf{f}} = M$.[34, 32] The $(1, d)$ entropy of a state $\mathbf{f}$ is given by,

$$S_{\mathbf{f}} = S_d(\mathbf{f}) = \sum_{k=1}^{s} e\,\Gamma(d+1, 1-\log f_k) - 1 \tag{4}$$

This is the correct entropy measure for systems where interactions between individual members create interdependences between individual states ensuring that $\Omega(N)$ takes the limiting form, $\lim_{N\to\infty} \frac{\Omega(N)}{\Omega'(N)} \log \Omega(N)^{\frac{1}{N}} = d$. In these systems, $d > 1$ is a multiplicative measure of the extent to which, asymptotically, the Shannon entropy overstates the rates at which their phase-space volumes grow over N. Interdependences ensure those rates of growth are slower by a factor d.

The persistent and ubiquitous ability of stretched-exponential functions (with d values well in excess of one) to describe the observed distributions of RoC suggest that the mathematical characteristics of statistical equilibria at stretched-exponential functions are the observational burden bearing upon any observationally successful systemic theorization of the competitive regulation of individual measures of profitability. Put differently, any such theory must be mathematically equivalent to two contentions defining those statistical equilibria.

First, that competition among enterprises in goods, labor, and capital markets create intractable, long-distance interdependences between their realized measures of profitability, ensuring $(1, d)$ entropy functionals offer the correct measures of phase-space volume in capital-market systems. This can be understood to reflect the fact that all economic outcomes for competing enterprises are generally dependent on each other, ensuring that not all conceivable micro-level configurations are in fact available to the economic system. Second, competition also ensures that the regulation of values of RoC toward the modal value r_d is mathematically equivalent to the presence of a constraint on the average deviation from that value in each tail.

In what follows, this paper develops a theoretical account of the competitive regulation of RoC that is mathematically equivalent to these two contentions. This formal account has very simple economic content—based on long-standing Classical and Austrian insights concerning the emergent outcomes of capital-market competition—and a very simple combinatorial motivation, based on a formal informational accounting of competitive outcomes.

3 Macroscopic Stability and Theorizing Competition

Before proceeding, a brief methodological discussion motivating and defining the application of information-theoretic tools for analysis of complex economic systems is necessary.

The persistence of a highly organized pattern in distributions of measures of profitability across countries with different competitive environments, regulatory practices and institutions, and accounting conventions is striking. Competition ensures individual values of profitability are interdependent. Locally, the effects of efforts by enterprises to boost profitability through technical innovation, through the development of new products, consumers, and markets, and through control over wages and other input costs are dynamically influenced by the analogous actions of other enterprises competitively engaged in the same input and output markets.[35, 36] More broadly, developed credit and capital markets create long-range interdependences between individual measures of profitability: They enable the general reallocation of capital value, productive capacities, and competitive efforts from low- to high-profitability undertakings, jointly conditioning demand and supply conditions, prices, and profitability across the markets involved.[35, 16, 18] Finally, at least over some time horizons constraints on aggregate demand for goods and services may create further long-range interdependences between measures of profitability.[37, 38, 39, 40]

The distributions we observe are thus generated by systems composed of very large numbers of non-linearly coupled members. Yet the complex micro-level competitive interactions involving enterprises, investors, workers, and consumers in each of these economies persistently resolve themselves into the simple distributional pattern reported above. The presence of significant measures of macroscopic stability across distributions of a quantity at the very heart of the functioning of decentralized capitalist economies has important implications for economic analysis.

3.1 The Method of Observational Political Economy

Contemporary mainline Economics seeks to characterize competitive processes on the basis of detailed descriptions of individual behavior.[41] Those descriptions are based on strong specifications of the consumption preferences, technological constraints, and knowledge states shaping individual actions, and of the forms taken by competitive interactions.[42] While potentially useful as bases for pursuing thought exercises, there are at least three reasons ensuring this approach offers a poor foundation for observational inquiry into the functioning of economic systems.[11]

First and as well established across a variety of disciplines, detailed descriptions of individual behavior are at best impractical bases to characterize the functioning of large, complex systems composed of many interacting parts. This is true even when the laws or regularities governing individual behavior are very well understood. Economic systems pose an additional and characteristic difficulty relative to physical systems in this regard: all economically relevant features of individuals are themselves shaped by economic competition and broader social interactions. If the characteristics of economic individuals and economic interactions are mutually defining, taking the former as an analytical starting point is not just impractical. It is arbitrary: There is no *a priori* reason to expect regularities allowing us to develop sucessful characterizations of the functioning of economic systems to be defined at the level of individual characteristics and behavior. In fact, contributions from a diverse range of traditions in political economy and economics have pointed to the role of structural relationships and systemic processes—like income-expenditure identities, competition, and input-output interrelationships—in defining important regularities we may be able to observe in economic systems.

Second, many of the individual characteristics and behaviors upon which contemporary theorizations of competition are predicated are unobservable,[36] straining the scientific soundness of suppositions made about their nature.[43] And third, the annual or quarterly frequencies at which we are typically able to observe some elements of individual economic states are far lower than the frequencies at which individuals interact. The quantities we can observe reflect not the behavior of individuals *per se*, but the accumulated result of many interactions among large numbers of individuals. Between the times at which we can take measurements, much of the information about individual behavior has been lost—both as a result of large numbers of interactions, and of changes to the individuals themselves.

Macroscopic stability in distributions of profitability enables the pursuit of a more fruitful analytical path. It strongly suggests the dynamic trajectories of individual measures of profitability are subordinated to a general, emergent pattern of systemic regulation common to all observed economies. Competition appears to impose upon decentralized capitalist economies a consistent form of the emergent self-organization or "spontaneous order" postulated by some of the salient figures in the history of economic thought.[16, 44, 45, 46] In the frequencies of profitability shown above we are in fact observing the manner and extent to which Adam Smith's "invisible hand" regulates the outcomes of individual competitive efforts. Those frequencies define the explanatory burden of observationally grounded, systemic theories of economic competition.

3.2 Existing Accounts of the Competitive Regulation of Profitability

The formal persistence in distributions of profitability has been the subject of two recent theorizations. Both characterize those distributions as statistical equilibria, but define them at least in part in relation to detailed descriptions of "representative" or average individual dynamic evolutions or individual cognitive constraints. Recourse to detailed descriptions of the characteristics of an individual in accounting for competitive processes creates a few difficulties.

The first contribution offered a drift-diffusion model of a "representative," time-homogeneous evolution for individual profitability capable of generating stationary Subbotin distributions.[8] The most significant difficulty with this line of explanation follows from the fact that economic competition is fundamentally about interactions, interdependences, and various dimensions of heterogeneity across competitors. A representation that reduces the outcomes of such processes to large numbers of identical, independent, individual evolutions may be formally successful, but it is not readily evident how those evolutions may be related to economic behavior or functioning.

The second contribution offers a more economically meaningful interpretation, based on the idea of "Quantal-Response Statistical Equilibria" (QRSE) recently developed and applied to distributions of profitability and measures of productivity growth.[9, 47] That account draws on Game Theory and relies on the idea that economic agents have "bounded rationality" or are otherwise "inattentive."[48, 49] Under this view, distributions of profitability are seen as reflecting the limited capacity for information processing of the typical or average individual, who can only respond probabilistically when considering whether to undertake a profitable arbitrage reallocation of capital. The profitability of an enterprise, in turn, is taken as statistically conditioned by whether it is experiencing entry or exit of competitors in the markets where it operates. The interplay between individual quantal responses to enter or exit a market, and the response of profitability to those decisions, are taken to define negative-feedback processes that tend to keep measures of profitability near r_g.

While such an approach can support useful thought exercises, the appeal to well-understood individual cognitive limits creates three related difficulties for observational inquiry. First and as noted above, it is not generally possible to recover details of individual stimulus-response behavior from the annual data we observe. It is possible to use the data we observe to infer parameter values in QRSE models of individual behavior and of the response of profitability to it, but those inferences will be inherently shaped by model specifications. Without this important qualification, the results of such inferences may be very difficult to interpret.

Second, competitive entry and exit into markets depend both on the intentions of arbitrageurs and on the degree to which capital markets accommodate and finance them. Only at capital-market equilibria are all intentions accommodated. The degree to which competitive entry increases competition in an industry hinges not on how many individuals choose to enter, but on the measure of productive and competitive resources allocated to that industry. That hinges most generally on the volume of financing would-be arbitrageurs can mobilize. A single, well-funded arbitrageur can have a more dramatic impact on competition and profitability than many poorly funded ones. This is closely related to a final difficulty. In competitive capital markets, individual cognitive limits are likely to be wholly irrelevant to systemic market outcomes. One person's error or broader inability to undertake an existing arbitrage operation is another person's opportunity to do so until the potential for arbitrage profits is exhausted.

The observed distributional patterns are not reducible to the intentions, knowledge, or cognitive limitations of any individual. Observable economic outcomes seldom are.[19] They are the emergent result of complex competitive interactions across all markets for goods, labor, and capital. The next two sections develop a formal, systemic model of the observable outcomes of competition that accounts for them based not on strong specifications of individual evolutions or behavioral characteristics but on the basis of rather simple, macroscopic economic and combinatorial considerations.

4 Competition, Arbitrage, and Organization

Competitive capitalist enterprises attempt to maximize returns on their assets. Their efforts to boost profitability are multifaceted, and involve intentions, actions, and results that are unobservable or at best difficult to observe. Enterprises may seek to develop and supply new products, to discover new production techniques, markets, or new ways to shape market or regulatory conditions. They may also move capital value and competitive efforts to existing undertakings enjoying comparatively high measures of profitability, or try to follow the productive, managerial, and marketing practices of more profitable firms. We term the former type of competitive intervention *entrepreneurial* or innovative and the latter emulative type *arbitrage*, since it generates profits from differentials in rates of return.

Neither enterprises nor economic observers can know what the effects of any given competitive action on measures of profitability will be. That depends on a complex range of interactions involving many enterprises, as well as broader changes to economic conditions arising independently from their actions. But the measure of profitability r_k realized by an enterprise relative to other realized measures of profitability offers an informative quantification of the extent to which the enterprise has been successful in its plans. It is a signal that guides further competitive actions.

As a result, the distribution of profitability does not simply reflect competitive actions. It also shapes them. *It is an observable social structure that plays a central functional role in competitive processes.* The persistent distributional patterns we observe across individual measures of profitability are thus evidence of spontaneous self-organization over a domain that is central to the functioning of decentralized, market economies. Their statistical and informational characteristics yield formal insights into emergent, macroscopic outcomes of competition in those economies.

This section discusses the propositions concerning the economic content of those characteristics that inform the formal model developed in the next section. It uses an *informational accounting* of the outcomes of complex patterns of economic competition to postulate a generalized, formal version of the Classical theory of the competitive regulation of profitability: That the pursuit of arbitrage profits is the agency responsible for the patterns of organization we observe in distributions of profitability. Those patterns may be understood to reflect two emergent economic consequences of that pursuit: The competitive aggregate tradeoff between gross returns and costs accruing to those pursuing arbitrage returns over any given period of time, and the manner in which the endogenous formation of the general or opportunity rate of return r_g reflects the presence of entrepreneurial or monopolistic quasi-rents. These contentions are considered in turn

4.1 An Informational Accounting of Competitive Processes

The strong peakedness in those distributions supports the Classical contention that, by shifting capital value, resources, and competitive efforts toward undertakings enjoying comparatively high measures of profitability, arbitrage tends to lower rates of return on those undertakings while potentially increasing those of previously less profitable ones now facing less competition. This tends to concentrate or organize measures of profitability amid broader, complex patterns of competitive interaction, giving rise to a general rate of return on capital, r_g. This rate emerges as the measure of the opportunity cost of capital—the rate investors come to regard as typical and demand on their investments. Deviations from this cost, $x_k = r_k - r_g$, offer a measure of *excess returns*. Its absolute value, $|x_k|$, offers a measure of the aggregate arbitrage profit that may be realized when a marginal unit of capital is profitably reallocated between a specific undertaking earning r_k and a typical one earning r_g. Most broadly, Classical Political Economy advanced that the pursuit of these profits that regulates distributions of RoC.

It is possible to generalize and formalize this contention with the help of an informational accounting of the competitive functioning of a decentralized market economy: A mapping from different types of competitive interactions and broader economic developments we may be able to characterize, to observable informational features of distributions of profitability.

Formally, let $\mathbb{E}$ be an exhaustive description of the state of an economic system. This state may be partially represented by distributions of profitability across its competitive enterprises $\mathbf{f}$, defined be the relative occupancy of all coarse-grained values of x_k. Those distributions are central to the functioning of market economies. They offer quantifications of the comparative success of different entrepreneurial undertakings, reflecting outcomes across all markets in the economy. They also inform future competitive actions, shaping the comparative measures to which different undertakings will be pursued.

The entropy $S_{\mathbf{f}}$ of any given distribution of profitability $\mathbf{f}$ can be usefully related to the competitive functioning of a complex economic system. The persistent observation that those distributions are remarkably well described by stretched-exponential functions strongly suggests that the processes generating them are non-multinomial, and that $(1, d)$ entropy functionals offer the correct informational measures of the phase-space volumes supporting any given macroscopic state or distribution. We consider that this reflects how competitive interactions in markets for goods, labor, and capital, create persistent interdependences between individual values of x_k that effectively reduce the measure of the overall phase spaces available to competitive economic systems. In this setting, d offers an informational measure of the interdependences between individual values of profitability established by the competitive functioning of a decentralized, market economy.

Explicit consideration of the dynamic evolution of $S_{\mathbf{f}}$ can cast further light on the evolution of a competitive economic system, $\mathbb{E}_t$. Formally,

$$\dot{S}_{\mathbf{f}}[t] = \dot{S}_{\iota}[t] - \dot{O}[t] \tag{5}$$

where $\dot{S}_{\iota}[t]$ represents the rate of gross entropy production, and $\dot{O}[t]$ represents the rate of gross entropy reduction taking place in the system.

While we cannot observe individual competitive actions and interactions, and competitive agents themselves cannot know what the impact of their interventions on their profitability and that of competitors will be, basic combinatorial considerations help cast light onto the systemic or macroscopic effects of different types of competitive actions. We know from Information Theory and Statistical Mechanics that the likelihood of "accidental" reductions in the entropy of $\mathbf{f}$ is overwhelmingly small for large-N systems. The only class of competitive interventions that can be statistically expected to reduce entropy are those driven by their central economic logic to do so: the large collection of individual arbitrage interventions by agents seeking the pecuniary returns that may be realized as long as there is heterogeneity or non-zero measures of entropy in the distribution of profitability.

We can reasonably expect other competitive actions and broader developments in the economy to result in gross increases in $S_{\mathbf{f}}$. It is thus plausible to suppose that gross reductions in the entropy of that distribution are an emergent, unintended outcome of large numbers of the pursuit of arbitrage profits. As complex competitive interactions drive the dynamic evolution of decentralized, market economies, it is the competitive mobility of capital value in pursuit of arbitrage returns that shapes the patterns of organization we observe in the central regions of distributions of profitability. To understand that shape, it is necessary to consider the economic calculus conditioning those actions, their outcomes and informational moments. Those will reflect two emergent economic realities.

4.2 Aggregate Returns and Costs of Arbitrage

We cannot observe individual arbitrage interventions or estimate the returns and costs they pose to those undertaking them. But we can reasonably suppose that in competitive capital markets populated by alert, profit-seeking agents, self-interest and price signals will tend to yield outcomes that maximize aggregate arbitrage profits.[36, 46, 50] Those are given by gross arbitrage returns net of the costs arbitrageurs incur in order to realize them. It is this emergent, competitive reckoning of aggregate arbitrage returns and costs that shapes the stretched-exponential distributions we observe.

To characterize this formally, consider a brief lapse of time $[0, \tau]$ during which competitive interactions take an economic system from a macroscopic state $\mathbb{E}_0$ containing an original profitability distribution $\mathbf{o}$ with entropy $S_{\mathbf{0}}$, to a final state $\mathbb{E}_\tau$ containing a distribution $\mathbf{f}$ whose entropy is $S_{\mathbf{f}}$. Integration of equation 5 assures us that the entropy change during any such time lapse is given by $S_{\mathbf{o}} - S_{\mathbf{f}} = \Delta S_\iota - \Delta O$. Put differently, the total gross entropy reduction effected by the actions of arbitrageurs over this period is given by,

$$\Delta O = (S_{\mathbf{o}} + \Delta S_\iota) - S_{\mathbf{f}} \tag{6}$$

Note that the quantity inside the parentheses in 6 can be understood as independent from arbitrage interventions and from the values taken by the system's distribution during the time period in question. $S_{\mathbf{o}}$ is given at the start of the period; and total entropy production is dominated by the effects of entrepreneurial discoveries and interventions—which proceed at their own idiosyncratic pace—and, perhaps most significantly, of exogenous changes in the conditions of the competitive system.

Whatever the details of the dynamic evolution of the competitive system over the brief time period in question, the total, aggregate measure of gross returns realized by arbitrageurs is overwhelmingly likely to vary negatively with $\langle|x|\rangle_{\mathbf{f}}$—the average *foregone* arbitrage returns latent in the end-of-period distribution $\mathbf{f}$. Competition can be understood to condition outcomes that minimize this moment of the distribution of profitability, given all other systemic considerations shaping the distribution of x_k.

A characterization of the aggregate costs posed by the pursuit of arbitrage costs over the period in question requires a more deliberate exposition. Those costs are defined economically by expenditures necessary to identify and pursue those actions, as well as by the risks they pose. Despite our inability to observe them, we can characterize their aggregate measure informationally, based on a simple conclusion: It is overwhelmingly likely that as the pursuit of arbitrage profits tends to minimize $\langle|x|\rangle$, the total costs of the large number of actions pursuing those profits are increasing on the gross entropy reductions those interventions effect on the distribution of profitability.

In considering this relationship it is important to note that the economic processes involved in the regulation of x_k in each tail of the overall distribution of RoC are well known to be different. Secondary markets for capital goods are notoriously problematic, typically creating stiff transactions costs for parties wishing to liquidate positions in fixed assets.[51] At the same time, competitive pressures are likely to be stronger in left tails since ongoing losses relative to r_g can result in the death of enterprises. The movement of capital into high-yielding undertakings may also pose costs of a distinctive nature, including as a result of strategic, deterrent actions by incumbents.[52, 53] This ensures that, in general, the costs and effects of arbitrage interventions, and the character of competitive interdependences between individual measures of x_k are different across each tail. As a result, it is necessary to consider the relationship between total costs of arbitrage interventions and the informational gains they bring about separately for each tail in the distribution.

Within each tail, total costs C will be given by the time integral of the instantaneous total arbitrage costs incurred at each $t \in [0, \tau]$. The latter, in turn, are given by the sum of the individual costs c_i of all arbitrage interventions undertaken under state $\mathbb{E}_t$, which we represent by a set of actions $A[\mathbb{E}_t]$ containing a large number $i = 1, ..., a[\mathbb{E}_t]$ of individual interventions. Formally,

$$C = \int_0^\tau \sum_{i=1}^{a[\mathbb{E}_t]} c_i [\mathbb{E}_t]\, dt = \int_0^\tau \bar{c}[\mathbb{E}_t]\, a[\mathbb{E}_t]\, dt \tag{7}$$

Where $\bar{c}[\mathbb{E}_t]$ denotes the average cost per arbitrage intervention undertaken at time t.

Along similar lines, the total gross entropy reduction taking place during this period, ΔO, is given by the integral of the instantaneous gross entropy reductions jointly effected by the actions in $A[\mathbb{E}_t]$. The latter reductions are the unintended consequence of the actions of arbitrageurs, and hinge on the state of the economy and on the set of arbitrage interventions in the tail in question during time t. Formally, denote this by $\dot{O}[t] = \dot{O}[\mathbb{E}_t, A_t]$, so that,

$$\Delta O = \int_0^\tau \dot{O}[\mathbb{E}_t, A_t]\, dt = \int_0^\tau \bar{o}[\mathbb{E}_t, A_t]\, a[\mathbb{E}_t]\, dt \tag{8}$$

Where $\bar{o}[\mathbb{E}_t, A_t]$ is the average effected gross entropy reduction per arbitrage intervention undertaken at time t.

It is clearly impossible to characterize C and ΔO fully. That would require detailed knowledge not only of the functions A, a, c_i, and $\dot{O}$, but of the entire path traced by $\mathbb{E}_t$ during the period in question. But so long as the mean average cost and mean average entropy reduction across all possible paths traced by $\mathbb{E}$ are well-defined and positive, the Law of Large Numbers ensures that it is overwhelmingly likely that higher measures of ΔO occur side-by-side with higher total pecuniary costs C—since both require a larger number of arbitrage interventions over the time period.

As a result, total costs of arbitrage interventions within a tail can be described by a cost function $C[\Delta O]$ with $C'[\Delta O] > 0$. We also suppose the function is either weakly convex or not "too" concave on ΔO that it becomes concave on $\mathbf{f}$, despite the convexity of $-S_{\mathbf{f}}$.

4.3 The Formation of r_g, Social Scaling, and Entrepreneurial Quasi-Rents

Since r_g emerges endogenously from competitive actions informed by comparative measures of individual profitability, its value hinges on the structure of individual measures of r_k. The set of realized rates of return influences what comes to be regarded as a typical rate of return. In Classical Political Economy, r_g was broadly understood as the "average" rate of profit.[17] Here we follow more recent contributions that have considered more general endogenous averaging processes in the formation of the opportunity cost of capital and how they may give rise to patterns of *social scaling* potentially defining simple regulations on distributions of profitability.[15, 11]

The dependence of r_g on individual measures of profitability in the economy creates complex interdependences between enterprises. In economic terms, it ensures the cost of capital an enterprise faces is competitively shaped by the rates of return realized by all other enterprises that may also be funded in capital markets. Individual values of x_k are interrelated, posing additional difficulties for analyses based on detailed descriptions of individual evolutions of values for that quantity. At the same time, those interdependences can ensure that measure of excess returns is a *socially scaled* variable. Many forms of social scaling can impose simple, macroscopic regulations across the distribution of x_k over certain time periods.

Economically speaking, the competitive formation of what capital-market actors come to regard as a typical rate of return will reflect the presence of dynamic *quasi-rents* or atypically high rates of return for some enterprises. Competitive processes see the recurring development of dynamic quasi-rents for a variety of reasons. Enterprises continuously develop advantages competitors may not readily reproduce,[54, 55, 56] like a corporation's accumulated knowledge, patents, brand name, or reputation,[57, 58] or its particular technological or organizational competences.[59] If successful innovations are difficult to emulate, competitive agents in capital markets may effectively regard the rates of return they generate as less representative of the general opportunities facing investors. A second possibility involves the presence of risk aversion. Undertakings with greater expected variability in their returns may attract less competition, resulting in comparatively higher average returns. As a result, we may expect more enterprises with riskier returns to be observed at higher measures of r_k. Finally, some enterprises may be able to establish monopolistic barriers on entry by competitors into some of the markets in which they engage. In all cases, some enterprises will exhibit measures of profitability that are understood as *atypically* high, on undertakings or that are not readily emulated by competitors, at least over some measures of time.

The presence of quasi-rents can ensure that the opportunity cost of capital is lower than the average of all measures of profitability in an economy. Statistically, this ensures that distributions of RoC have right skews. The influence of quasi-rents in the formation of r_g may be formalized in a variety of ways congruous with those skews. Here we consider a simple possibility that lends itself readily to economic interpretation.

Suppose that along generalized Classical lines the general rate of return r_g is simply a weighted average $\langle ar \rangle_{\mathbf{f_s}}$ of individual rates of return r_k, with normalized weights $\langle a \rangle = 1$, where $\mathbf{f_s}$ is the distribution of profitability over values greater than r_g. In this setting, the measure of excess returns x_k we are considering is given by $x_k = r_k - \langle ar \rangle_{\mathbf{f_s}}$, which ensures that,

$$\langle x \rangle_{\mathbf{f_s}} = - \langle a, r \rangle_{\mathbf{f_s}} \equiv b \tag{9}$$

The presence of entrepreneurial quasi-rents would ensure this covariance is negative—higher rates of return are deemed to be less representative or "typical" of rates of return available to investors, resulting in $b < 0$. We suppose that over short periods of time, capital-market competition ensures there is stability in the association between the measure of quasi-rents enjoyed by enterprises and the extent to which those returns are considered "typical". This may ensure that over those periods $\langle a, r \rangle_{\mathbf{f_s}}$ is stable, and that b too is stable over those periods.

This last condition will be shown below impose the observed right skews in distributions of profitability during the complex, competitive regulation of profitability by the pursuit of arbitrage profits.

5 The Systemic Model

We now characterize formally the end-of-period distributions $\mathbf{f}$ as the systemic, macroscopic result of the competitive pursuit of arbitrage profits over the time period in question. We suppose that competition creates tendencies for the exercise of arbitrage to result in a reckoning of gross returns arbitrageurs realize over the period in question with the gross aggregate costs they incur. All those seeking arbitrage profits raise funds or have obligations to the same capital market. The competitive capital-market prices (or rates of return demanded by liability holders) they face will tend to reflect the demand by all of them for funds to support their actions. Competitive capital-market processes thus tend to result in the undertaking of arbitrage operations that maximize net aggregate arbitrage returns or, conversely, minimize aggregate arbitrage losses.

In each tail of the distribution, aggregate arbitrage costs are taken as rising in the measure of informational gains achieved during the time period $[0, \tau]$. Differences in the economic processes responsible for the regulation in each tail ensure that the aggregate arbitrage cost functions in each tail are in general different, and that the parameter d in the $(1, d)$ entropy functional describing the multiplicity of each tail distribution is generally different as well. Formally, this means those costs can be expressed in relation to the end-of-period distribution $\mathbf{f_n}$ over infra-modal bins x_i, and to distributions $\mathbf{f_s}$ over supra-modal bins x_j, according to the increasing, convex functions,

$$C_n = C_n \left[\Theta_n - S_{d_n}(\mathbf{f_n})\right] \quad ; \quad C_s = C_s \left[\Theta_s - S_{d_s}(\mathbf{f_s})\right] \tag{10}$$

where in each tail, $\Theta_i = S_0^i + \Delta S_\iota^i$ denotes the sum of the start-of-period entropy and the entropy created during the time period.

We suppose that during $theperiod\,[0, \tau]$ the measure of quasi-rents b is constant, ensuring that the constraint in **??** holds. Finally, we also suppose that during that period, as the actions of arbitrageurs tend to bring measures of x_i and x_j toward zero, they do so without significantly changing the overall distribution of enterprises between left and right tails of the distribution. The share $a \in [0, 1]$ of enterprises with $x_k \geq 0$, and the share $(1 - a)$ of those with $x_k < 0$ is understood as determined by other interventions and economic developments, and taken as given to the economic processes regulating the aggregate results of arbitrage interventions during $[0, \tau]$.

The distributions we observe may be thus understood as the result of the following piecewise optimization problem, defined over the partial frequency $\mathbf{f_n}$ describing the relative occupancy of x_i bins, and the partial frequency $\mathbf{f_s}$ describing the occupancy all bin values x_j, with $\mathbf{f} = \mathbf{f_n} \bigcup \mathbf{f_s}$,

$$\begin{aligned} \min_{\mathbf{f_n}, \mathbf{f_s}} \langle |x| \rangle_{\mathbf{f}} + C_n \left[\Theta_n - S_{d_n}(\mathbf{f_n})\right] + C_s \left[\Theta_s - S_{d_s}(\mathbf{f_s})\right] \\ \text{subject to } \langle x \rangle_{\mathbf{f_s}} = b, \\ ||\mathbf{f_n}|| = (1 - a)\,, \;\; ||\mathbf{f_s}|| = a \end{aligned} \tag{11}$$

Where a, Θ_n, and Θ_s are given to the process. The associated Lagrangian for this problem is,

$$\begin{aligned}\mathbb{L} = \langle |x| \rangle_{\mathbf{f}} + C_n \left[\Theta_n - S_{d_n}(\mathbf{f_n})\right] + C_s \left[\Theta_s - S_{d_s}(\mathbf{f_s})\right] - \gamma \left(\langle x \rangle_{\mathbf{f_s}} - b\right) \\ + m_n \left(||\mathbf{f_n}|| - (1-a)\right) + m_s \left(||\mathbf{f_s}|| - a\right)\end{aligned} \tag{12}$$

where all $m_i > 0$ and the Lagrange multiplier $\gamma \in [-1, 1]$ measures the aggregate tradeoff between quasi-rents enjoyed by incumbents in undertakings yielding $x_k > 0$ and the net profits realized by those moving capital value toward such undertakings.

Optimization of this Lagrangian yields a piecewise optimal distribution,

$$f_i = \exp\left[1 + \left(\frac{x_i - m_n}{\delta_n}\right)^{\frac{1}{d_n}}\right] \quad ; \quad f_j = \exp\left[1 - \left(\frac{(1-\gamma)\, x_j + m_s}{\delta_s}\right)^{\frac{1}{d_s}}\right] \tag{13}$$

Where $\delta_n = C_n' \left(\Theta_n - S_{d_n}(\mathbf{f_n^*})\right)$ and $\delta_s = C_s' \left(\Theta_s - S_{d_s}(\mathbf{f_s^*})\right)$ are the marginal costs of additional informational gains in each tail at the optimal $\mathbf{f}^*$. These two functions are also subject to their respective normalization constraints, which are met when,

$$\frac{1}{\delta_n} = \frac{d_n e}{1-a} \Gamma\left(d_n, \frac{m_n}{\delta_n}^{\frac{1}{d_n}}\right) \quad ; \quad \frac{1-\gamma}{\delta_s} = \frac{d_s e}{a} \Gamma\left(d_s, \frac{m_s}{\delta_s}^{\frac{1}{d_s}}\right) \tag{14}$$

The right tail is also subject to the constraint on its expected value, $\langle x \rangle_{\mathbf{f_s}} = b$, from which it is possible to define γ as an implicit function of all other parameters in 12, with a positive derivative with respect to b.

It is trivial to show that these two solutions are equivalent to 1 and 2, with,

$$\frac{1}{\lambda_n} = \delta_n \equiv \kappa_n, \quad \frac{m_n}{\delta_n} = \mu_n \quad ; \quad \frac{1}{\lambda_s} = \frac{\delta_s}{1 - \gamma\,(b)} \equiv \kappa_s, \quad \frac{m_s}{\delta_s} = \mu_s \tag{15}$$

where κ_i represent the effective marginal aggregate cost of informational gains within tail i in the distribution. It is conditioned by the marginal expenditures arbitrageurs needed to undertake to increase the measure of organization in that section of the distribution, and—in the right tails—by the marginal losses would-be arbitrageurs face as a result of quasi-rents enjoyed by incumbents in high-return projects.

The model in equations 13 and 14 accounts for double stretched-exponential distributions of RoC neither as the result of would-be "representative," independent drift-diffusion evolutions, nor as expressions of the cognitive or broader characteristics of any individual. It does so based on a systemic characterization of the competitive, aggregate outcome of profit-seeking arbitrage interventions over any given lapse of time. The distributions are thus taken to reflect the economic calculus capital-market competition imposes on the set of all agents capable to undertaking emulative or arbitrage interventions. They are the outcome of the aggregate minimization of losses associated with the profit-seeking arbitrage, and the losses imposed on them by the presence of rents and dynamic quasi-rents among enterprises earning high returns.

The model defines a series of macroscopic measures that characterize different systemic features of competitive systems. The parameters d_n and d_s offer purely informational measures of the effects of general competitive interdependences on values of profitability; b offers a systemic measure of the significance of rents and quasi-rents in the economy; and κ_n and κ_s represent the effective pricing of informational gains arising from the competitive actions of arbitrageurs in the presence of those rents and quasi-rents. These measures may help inform the understanding of the systemic effects of competition and the formulation of competition policy.

6 Implications and Discussion

The model's success also points to at least three promising lines of further theoretical work.

The first relates to the broader applicability of the informational characterization of aggregate competitive outcomes offered by the paper. The approach to the systemic regulation of profitability may have general applicability to other quantities subject to regulation by arbitrage. An interesting instance that supports the approach taken above involves distributions of "Tobin's q," a measure of the valuation of a corporation's liabilities or securities relative to measures of the value of its assets. Those distributions are strongly organized around modal values. The simplest expression of that organization is the Asymmetric Laplace form it takes for the logarithm of q.[28] Notably, the difference between the logarithm of a corporation's q and the logarithm of the q value for the "typical" or modal corporation is a measure of pecuniary returns on arbitrage; in this case the capital gains a corporation's managers can realize by arbitraging between the effective cost of capital (given by the total rate of return on assets investors expect of the "typical" corporation) and the total rate of return on assets investors expect on its assets.[15] Here too, the Asymmetric Laplace distribution may be taken as conditioned by the aggregate tradeoff between gross arbitrage profits and the costs imposed by organization in the distribution of $\log q$. Its shape expresses the effective pricing of information arising from the competitive actions of arbitrageurs. This type of pricing may be a general feature of quantities regulated by competitive arbitrage, and of the empirical manifestation of the Law of One Price.

The second relates to the fact that the problem in 12 has a dual entropy-maximization presentation, under which the stretched-exponential distributions derived above can be understood as statistical equilibria. This opens the way for a new and distinctively Classical understanding of the content and competitive determination of prices.

All traditions in economic analysis agree that prices are most immediately conditioned by the interaction between supply and demand flows for the goods in question. Most of them predicate "natural" prices on sets of deterministic market equilibria. Where they generally differ is in their understanding of what ultimately conditions supply and demand for goods, which reflects different appreciations of the most general determinants and the content of equilibrium prices. Contemporary microeconomics predicates competitive prices on Walrasian general equilibria.[42] The supply and demand behavior defining those equilibria are in turn defined by strongly specified descriptions of the technological constraints facing individual firms and of the subjective preferences of individual consumers over bundles of goods. But production techniques and consumer preferences are continuously shaped and redefined by the competitive efforts of enterprises, posing significant conceptual problems for this parametrization of the competitive process: It predicates prices established by competition on individual characteristics that are themselves evolving as part competitive processes.[36]

Classical Political Economy takes a different approach. Because profits are most immediately defined by an enterprise's ability to sell their outputs at prices that exceed their expenditures on labor and on inputs purchased from other enterprises, it is possible to think of prices as part of economy-wide structures of "prices of production": Prices predicated on wage structures, input-output productive relationships, and on the measures of profitability they define for each enterprise.[35, 60] This approach is more general than Walrasian ones because it considers that the mobility of capital and broader competitive efforts regulate the evolution of supply and demand flows, giving rise to prices that equate measures of profitability across all undertakings in the economy. Unfortunately, this containts the implausible assumption of persistent, deterministic capital-market equilibria, creating serious conceptual and empirical difficulties.[2, 5, 61]

The findings reported here point to the usefulness of a *generalized* Classical approach that looks to the mobility of capital and broader competitive efforts as the most general determinant of the evolution of the structure of generalized prices of production in a capitalist economy: The dynamic evolution of those structures is shaped by a broad range of changes affecting an economy, including entrepreneurial efforts to innovate production techniques and to change preferences, market shares, and other conditions in input and output markets. But the observed outcomes of those complex competitive interactions also reflect the movement of capital and broader competitive efforts to higher yielding undertakings. This competitive movement exerts a general and persistent influence on supply and demand conditions across most markets, shaping the structure of prices. Those structures necessarily reflect the persistent macroscopic regularities in the distributions of profitability. The relationship between competitive price structures, wages, and productive techniques is mediated by the capital-market statistical equilibria documented by this article. The ensuing relationships hold independently of whether or not markets for goods are at equilibrium. This conclusion can usefully inform further observational work into the functioning and distributional content of decentralized capitalist economies.

Finally, the model proposed above may have wider application to analysis of a set of social and broader systems capable of self-organization over some of their functional domains. When considering the functioning of many such systems we know that entropy reductions are costly, in the sense that they are not spontaneous and require some form of input. But we often do not have anything like the clear relationship that exists between entropy reduction and its minimum energy input costs in thermodynamic systems.[62] Despite this, the paper has shown that if gross entropy reductions in distributions representing a system's state over some of its functional domains are the result of large numbers of individual processes, there are fairly general conditions that ensure a weaker result holds: That over any given time period, greater measures of gross entropy reduction have greater input requirements. This applies to any such system, no matter the nature of its functioning, of the processes reducing its entropy, or the kind of input needed to achieve entropy reductions. It is simply a statement that across thermodynamic, social, or cognitive systems, we should expect the rejection of greater volumes of their available phase spaces to require greater aggregate measures of the relevant input.

This parsimonious conclusion can be very informative in analysis of social, economic, and cognitive systems, whose functioning can often be characterized in relation to the effective pursuit of objectives or goals. In many instances, those goals may be characterized as the minimization of a moment of frequency distributions describing part of the macroscopic state of the system in question. Such minimizations require entropy reductions, creating a tension and tradeoffs between pursuit of the system's objective and its informational costs. Informational costs may have a variety of different expressions. They may appear as a constraint on measures of informational association between distributions representing a system's inputs and outputs.[49] They may appear as a lower bound on the entropy of outcomes of individual cognitive processes.[63, 9] Or they may appear as an explicit, positive schedule of costs posed by different measures of organization in a system's state—*inter alia*.

In all cases, the resulting tradeoffs will shape potentially observable distributions describing the macroscopic state of those systems over the domains in question. As in the present case, the observed distributions express a rather simple result—an effective pricing by the system in question of informational gains, measured in terms of the quantities defining its effective goals. This pricing reflects the influence of basic combinatorial or informational realities in the determination of all manner of systemic outcomes.

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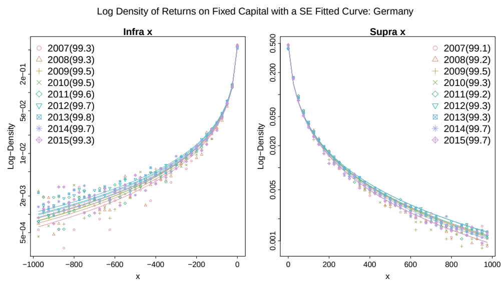
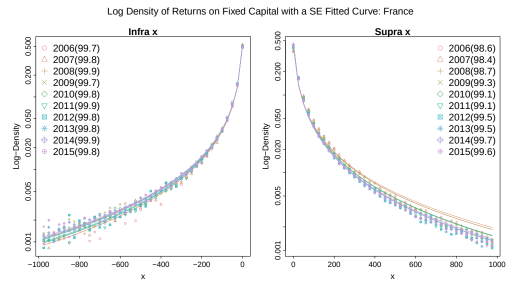
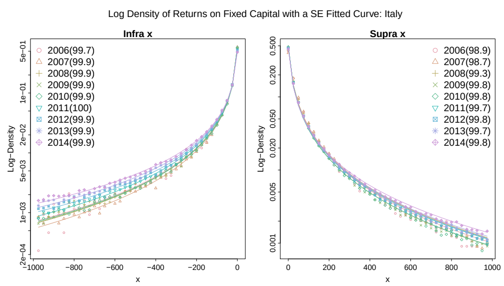
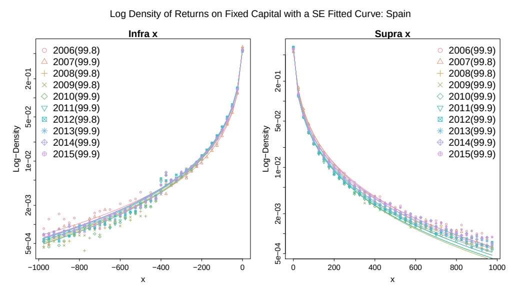
7 Appendix — Full Reports on Data and Fits

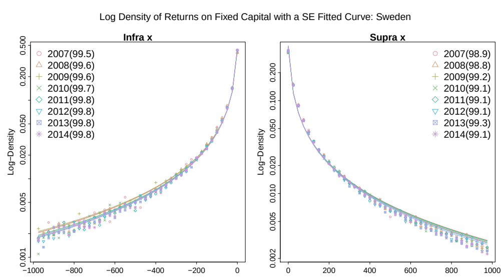
7.1 All Country Histograms and Fits

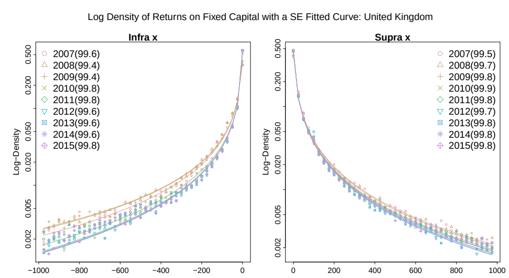


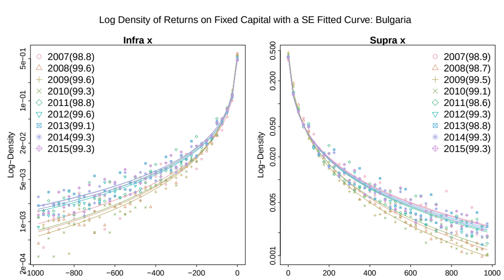


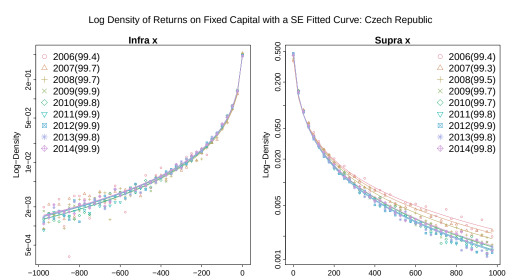




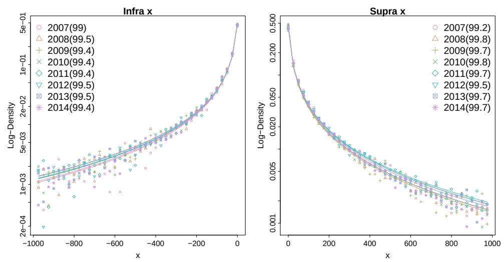




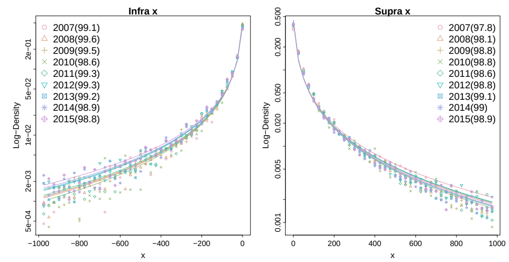




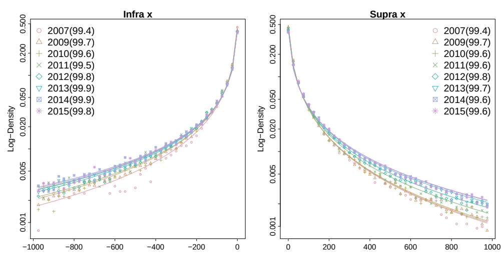
Log Density of Returns on Fixed Capital with a SE Fitted Curve: Estonia



Log Density of Returns on Fixed Capital with a SE Fitted Curve: Finland



Log Density of Returns on Fixed Capital with a SE Fitted Curve: Hungary



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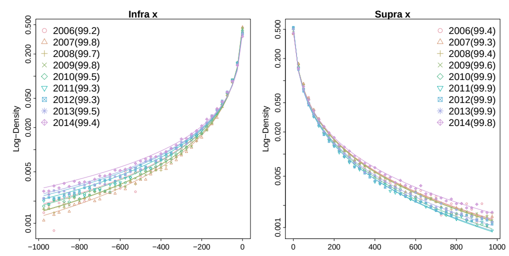

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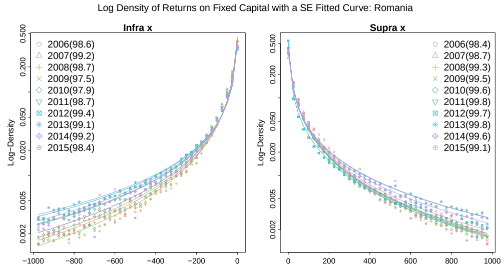

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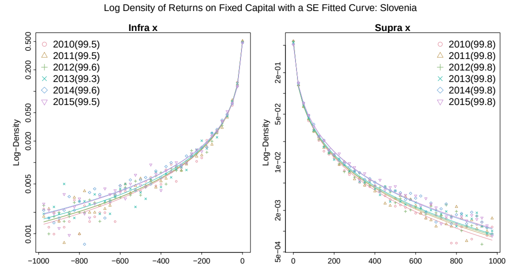

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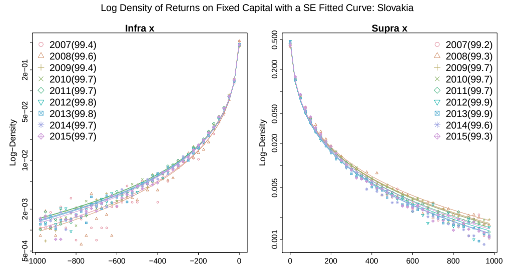
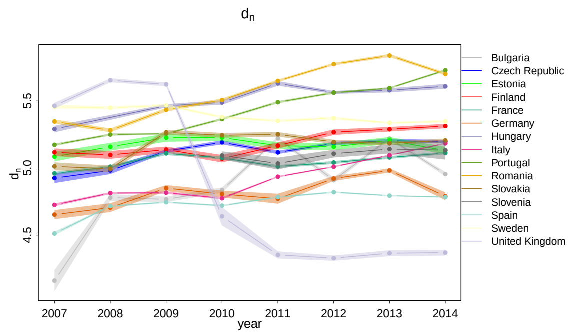
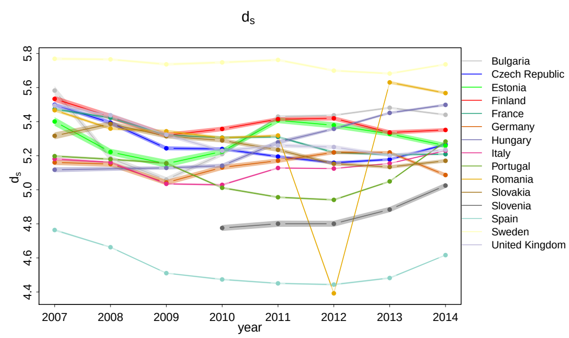
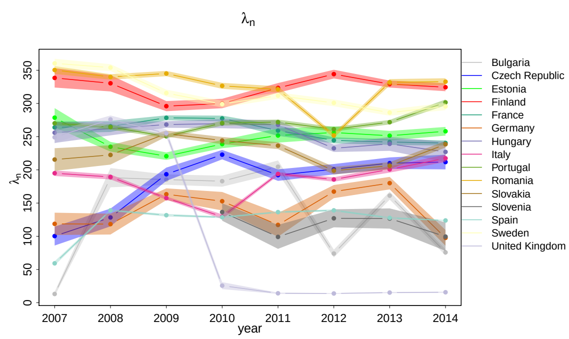
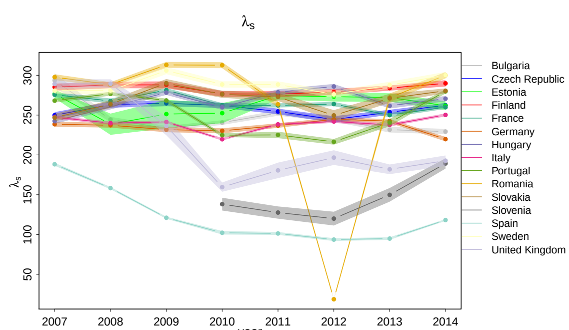


7.2 Estimated Parameter Values for All Country-Years









7.3 Goodness of Fit Statistics

Country	Year	Parameter d		Parameterλ		Soofi ID		Diff of ID with GN		Likelihood Ratio SE-GN	
		left tail	right tail	left tail	right tail	left tail	right tail	left tail	right tail	left tail	right tail
Bulgaria	2007	4.16 (0.08)	5.58 (0.03)	-13.2 (3.1)	293.5 (9)	98.82	98.88	0.02	0.11	1.000	1.001
	2008	4.78 (0.03)	5.18 (0.02)	-188.2 (14.5)	244.4 (3.3)	99.59	98.68	0.49	0.20	1.005	1.002
	2009	4.77 (0.02)	5.06 (0.01)	-185.3 (7.8)	233.6 (2.8)	99.57	99.45	0.77	0.82	1.008	1.008
	2010	4.84 (0.02)	5.21 (0.01)	-183 (8.3)	240.6 (3)	99.28	99.15	0.38	0.63	1.004	1.006
	2011	5.22 (0.01)	5.43 (0.01)	-204.7 (9.5)	252.6 (3.4)	98.79	98.56	0.09	0.26	1.001	1.003
	2012	4.91 (0.02)	5.44 (0.01)	-74 (5.9)	244.7 (3)	99.58	99.28	0.08	0.34	1.001	1.004
	2013	5.22 (0.01)	5.48 (0.01)	-161.2 (6)	231.7 (3.6)	99.07	98.78	0.27	0.26	1.003	1.003
	2014	4.96 (0.02)	5.44 (0.01)	-75.8 (4.5)	229.3 (3.3)	99.31	99.28	0.21	0.28	1.002	1.003
	2015	5.23 (0.01)	5.52 (0.01)	-182.9 (6.2)	231.6 (3.4)	99.26	99.31	0.16	0.17	1.002	1.002
Czech Rep.	2006	5.12 (0.05)	5.58 (0.02)	-178.8 (28.8)	266.5 (5.8)	99.42	99.37	0.32	0.12	1.003	1.001
	2007	4.92 (0.04)	5.5 (0.01)	-100.3 (14.3)	249.8 (3.9)	99.73	99.33	0.33	0.23	1.004	1.002
	2008	4.98 (0.03)	5.39 (0.01)	-128.4 (13.4)	263.1 (3.8)	99.70	99.53	0.40	0.18	1.004	1.002
	2009	5.13 (0.01)	5.24 (0.01)	-193.4 (10)	264.8 (3.6)	99.86	99.75	0.46	0.41	1.005	1.004
	2010	5.19 (0.02)	5.24 (0.01)	-222.9 (7.1)	262.2 (3.3)	99.84	99.70	0.14	0.40	1.002	1.004
	2011	5.12 (0.01)	5.19 (0.01)	-192.1 (8.7)	254.6 (3.5)	99.91	99.81	0.21	0.34	1.002	1.003
	2012	5.18 (0.01)	5.16 (0.01)	-201 (7.8)	244.4 (4.2)	99.87	99.87	0.17	0.36	1.002	1.004
	2013	5.2 (0.01)	5.18 (0.01)	-210.1 (8.7)	253.6 (3)	99.84	99.84	0.24	0.41	1.003	1.004
	2014	5.2 (0.01)	5.27 (0.01)	-211.8 (11)	262.3 (3)	99.87	99.75	0.27	0.27	1.002	1.003
Estonia	2007	5.08 (0.04)	5.4 (0.03)	-278.6 (14.4)	276.6 (5.3)	98.96	99.17	0.26	-0.02	1.002	1.000
	2008	5.16 (0.03)	5.22 (0.02)	-234.8 (7.3)	238.5 (13.3)	99.46	99.83	0.36	0.27	1.004	1.003
	2009	5.23 (0.03)	5.15 (0.02)	-220.6 (5.7)	251.3 (17.8)	99.36	99.67	0.16	0.23	1.002	1.002
	2010	5.23 (0.03)	5.22 (0.02)	-238.8 (6.2)	252.4 (12.7)	99.42	99.85	0.22	0.21	1.002	1.002
	2011	5.17 (0.03)	5.41 (0.02)	-251.7 (10.7)	274.5 (5)	99.41	99.71	0.41	0.27	1.004	1.003
	2012	5.16 (0.03)	5.38 (0.02)	-256.2 (9.3)	273.1 (6.1)	99.46	99.54	0.26	0.03	1.003	1.000
	2013	5.21 (0.03)	5.33 (0.02)	-251.3 (7.3)	272 (5.3)	99.47	99.65	0.07	0.25	1.001	1.002
	2014	5.13 (0.03)	5.26 (0.02)	-258.1 (6.5)	261.9 (5.5)	99.39	99.72	0.19	0.15	1.002	1.002
Finland	2007	5.12 (0.03)	5.53 (0.02)	-338.4 (14.5)	285.2 (4.3)	99.13	97.80	0.43	-1.01	1.004	0.990
	2008	5.1 (0.03)	5.43 (0.02)	-330.4 (12.3)	287.9 (4.1)	99.56	98.09	0.26	-0.71	1.003	0.993
	2009	5.14 (0.02)	5.32 (0.02)	-295.9 (7.6)	287.9 (4.4)	99.52	98.81	0.22	-0.04	1.003	1.000
	2010	5.07 (0.03)	5.36 (0.01)	-299.9 (7.1)	276.6 (3.7)	98.59	98.79	0.29	0.04	1.003	1.000
	2011	5.17 (0.02)	5.41 (0.01)	-323.4 (7.3)	276.4 (3.9)	99.34	98.59	0.04	-0.43	1.000	0.996
	2012	5.27 (0.02)	5.42 (0.01)	-344.1 (6.6)	279.1 (3.8)	99.28	98.84	-0.02	-0.20	1.000	0.998
	2013	5.29 (0.02)	5.34 (0.01)	-329.2 (5.3)	283.9 (3.2)	99.16	99.09	0.06	0.03	1.001	1.000
	2014	5.31 (0.02)	5.35 (0.01)	-324.4 (6)	289.9 (4)	98.87	98.98	0.07	0.23	1.000	1.002
	2015	5.38 (0.02)	5.39 (0.01)	-353.3 (5.6)	295.8 (3.5)	98.82	98.94	-0.28	0.15	0.998	1.002

Table 2: Estimated Parameters for Stretched Exponential and Generalized Normal Distributions for Bulgaria, Czech Republic, Estonia, and Finland.

Country	Year	Parameter d		Parameterλ		Soofi ID		Diff of ID with GN		Likelihood Ratio SE-GN	
		left tail	right tail	left tail	right tail	left tail	right tail	left tail	right tail	left tail	right tail
France	2006	4.97 (0.02)	5.43 (0.01)	-262.3 (11.5)	276.8 (3.6)	99.75	98.58	0.65	-0.76	1.006	0.992
	2007	4.96 (0.01)	5.47 (0.01)	-263.9 (8.8)	275.6 (3.1)	99.84	98.35	0.54	-0.98	1.006	0.990
	2008	5.01 (0.01)	5.42 (0.01)	-265.4 (8.4)	267.7 (3.1)	99.86	98.75	0.56	-0.62	1.006	0.994
	2009	5.11 (0.01)	5.32 (0.01)	-278.4 (3.7)	281.3 (2.6)	99.75	99.25	0.55	-0.03	1.006	1.000
	2010	5.08 (0.01)	5.31 (0.01)	-277.4 (4.3)	262.4 (2.7)	99.79	99.08	0.49	-0.13	1.005	0.999
	2011	5.01 (0.01)	5.31 (0.01)	-259.1 (7.9)	262.2 (2.6)	99.88	99.11	0.58	-0.10	1.005	0.999
	2012	5.04 (0.01)	5.22 (0.01)	-244.4 (4.5)	263.9 (2.7)	99.77	99.46	0.57	0.18	1.006	1.002
	2013	5.08 (0.01)	5.21 (0.01)	-241.9 (4.4)	250 (2.7)	99.80	99.51	0.60	0.29	1.006	1.003
	2014	5.11 (0.01)	5.21 (0.01)	-240.4 (3.9)	259.9 (2.2)	99.85	99.69	0.55	0.40	1.006	1.004
	2015	5.07 (0.01)	5.25 (0.01)	-228.7 (7.5)	267.1 (2.4)	99.84	99.61	0.64	0.26	1.006	1.003
Germany	2007	4.65 (0.03)	5.16 (0.02)	-118.7 (16.8)	238.5 (3.1)	99.34	99.09	0.54	0.08	1.006	1.001
	2008	4.71 (0.03)	5.15 (0.02)	-118.4 (15.7)	237.3 (3)	99.33	99.22	0.53	0.09	1.006	1.001
	2009	4.85 (0.02)	5.04 (0.02)	-163.3 (8.9)	231.7 (3.3)	99.49	99.54	0.59	0.49	1.006	1.005
	2010	4.81 (0.03)	5.13 (0.01)	-152.8 (13.9)	230.2 (3.2)	99.55	99.33	0.55	0.44	1.005	1.004
	2011	4.77 (0.04)	5.17 (0.01)	-117.1 (17.8)	236.8 (2.9)	99.64	99.19	0.34	0.28	1.003	1.003
	2012	4.92 (0.02)	5.22 (0.01)	-167.3 (9.9)	244.6 (2.4)	99.75	99.31	0.55	0.21	1.005	1.002
	2013	4.98 (0.02)	5.22 (0.01)	-180 (9.4)	242.5 (2.5)	99.76	99.29	0.56	0.17	1.006	1.002
	2014	4.79 (0.03)	5.09 (0.01)	-97.6 (11.8)	219.8 (2.8)	99.74	99.68	0.44	0.65	1.005	1.007
	2015	4.71 (0.04)	5.04 (0.01)	-84 (15)	219.3 (2.8)	99.34	99.67	0.54	0.79	1.006	1.008
Hungary	2007	5.29 (0.02)	5.12 (0.01)	-255.9 (15.4)	242.5 (3.6)	99.39	99.39	0.69	0.46	1.006	1.005
	2009	5.46 (0.01)	5.13 (0.01)	-268.2 (4.9)	278.2 (2.9)	99.68	99.66	0.18	0.56	1.002	1.006
	2010	5.49 (0.02)	5.14 (0.01)	-269.6 (6.9)	259.9 (3.5)	99.58	99.64	0.08	0.60	1.001	1.006
	2011	5.63 (0.02)	5.28 (0.01)	-267.8 (6.8)	278.7 (3.7)	99.49	99.58	-0.01	0.34	1.000	1.003
	2012	5.56 (0.01)	5.36 (0.01)	-232.4 (5.5)	285.9 (3.2)	99.79	99.77	0.09	0.38	1.001	1.004
	2013	5.58 (0.02)	5.45 (0.01)	-239.1 (10.5)	261.7 (3.4)	99.87	99.68	0.17	0.24	1.002	1.002
	2014	5.61 (0.02)	5.5 (0.01)	-227 (15.3)	270.6 (3.4)	99.87	99.64	0.27	0.23	1.002	1.002
	2015	5.62 (0.02)	5.51 (0.01)	-258.6 (11.3)	264.1 (3.6)	99.81	99.57	0.21	0.11	1.002	1.001
Italy	2006	4.84 (0.03)	5.11 (0.02)	-185.2 (11.1)	247.7 (4)	99.66	98.87	0.46	-0.07	1.005	0.999
	2007	4.73 (0.01)	5.18 (0.01)	-194.9 (4.2)	247.3 (1.9)	99.90	98.73	0.40	-0.41	1.004	0.996
	2008	4.81 (0.01)	5.16 (0.01)	-189.3 (3.8)	239.9 (2)	99.91	99.30	0.51	-0.08	1.005	0.999
	2009	4.82 (0.01)	5.03 (0.01)	-157.5 (2.4)	241.4 (1.6)	99.94	99.76	0.44	0.33	1.004	1.003
	2010	4.77 (0.01)	5.03 (0.01)	-129.5 (3.6)	219.7 (1.6)	99.95	99.76	0.45	0.33	1.004	1.003
	2011	4.94 (0.01)	5.13 (0)	-193.9 (2.6)	238.4 (1.7)	99.96	99.68	0.46	0.29	1.005	1.003
	2012	5.01 (0.01)	5.12 (0)	-185.5 (3.3)	241.9 (1.8)	99.95	99.78	0.45	0.35	1.004	1.004
	2013	5.1 (0.01)	5.15 (0)	-200.7 (4.9)	237.7 (2)	99.94	99.74	0.44	0.33	1.005	1.003
	2014	5.18 (0)	5.23 (0)	-217.4 (5.5)	250.2 (1.9)	99.94	99.77	0.44	0.36	1.004	1.004

Table 3: Estimated Parameters for Stretched Exponential and Generalized Normal Distributions for France, Germany, Hungary, and Italy.

Country	Year	Parameter d		Parameterλ		Soofi ID		Diff of ID with GN		Likelihood Ratio SE-GN	
		left tail	right tail	left tail	right tail	left tail	right tail	left tail	right tail	left tail	right tail
Portugal	2006	5.4 (0.02)	5.21 (0.02)	-302.6 (5.8)	290 (4.6)	99.23	99.38	0.23	0.43	1.002	1.004
	2007	5.17 (0.01)	5.2 (0.01)	-270.1 (2.8)	268.2 (2.5)	99.79	99.30	0.49	0.35	1.005	1.004
	2008	5.25 (0.01)	5.18 (0.01)	-264.4 (3.6)	276.8 (2.4)	99.69	99.42	0.49	0.50	1.005	1.005
	2009	5.26 (0.01)	5.16 (0.01)	-250.5 (2.9)	268.3 (2.4)	99.75	99.62	0.55	0.51	1.005	1.005
	2010	5.36 (0.01)	5.01 (0.01)	-270.4 (3.7)	224.8 (2.3)	99.51	99.86	0.31	0.61	1.003	1.006
	2011	5.49 (0.01)	4.96 (0.01)	-271.9 (3.8)	225.1 (4)	99.26	99.90	-0.04	0.64	0.999	1.006
	2012	5.56 (0.01)	4.94 (0.01)	-261.2 (3.5)	216.4 (3.6)	99.28	99.93	-0.02	0.56	0.999	1.006
	2013	5.6 (0.01)	5.05 (0.01)	-271.8 (3.4)	240.6 (4.7)	99.47	99.93	0.07	0.53	1.000	1.005
	2014	5.73 (0.01)	5.28 (0.01)	-301.6 (4.4)	279.8 (2.6)	99.41	99.80	0.01	0.45	1.000	1.004
Romania	2006	5.65 (0.03)	5.73 (0.02)	-357.1 (9.5)	347.6 (7.3)	98.62	98.42	-0.18	-0.96	0.998	0.990
	2007	5.35 (0.02)	5.47 (0.01)	-350.3 (5.9)	297.7 (3.6)	99.22	98.72	0.42	-0.73	1.004	0.993
	2008	5.28 (0.01)	5.36 (0.01)	-339.9 (4.1)	288.3 (3)	98.74	99.29	0.34	-0.07	1.003	0.999
	2009	5.43 (0.01)	5.34 (0.01)	-345 (4)	313 (3.8)	97.53	99.53	-0.97	0.37	0.990	1.004
	2010	5.51 (0.01)	5.31 (0.01)	-326.5 (4.5)	312.7 (3.8)	97.87	99.64	-1.23	0.58	0.988	1.006
	2011	5.65 (0.01)	5.32 (0.01)	-321.2 (4.6)	263.6 (4.2)	98.73	99.83	-0.47	0.46	0.995	1.005
	2012	5.77 (0.01)	4.39 (0.01)	-251.7 (3.5)	18.8 (0.4)	99.45	99.73	-0.05	-0.07	1.000	0.999
	2013	5.84 (0.01)	5.63 (0.01)	-331.8 (4.8)	271.8 (5.3)	99.05	99.78	-0.05	0.46	1.000	1.005
	2014	5.7 (0.01)	5.57 (0.01)	-332.8 (5.1)	300 (4)	99.24	99.58	0.04	0.27	1.000	1.003
	2015	5.5 (0.02)	5.4 (0.01)	-353.2 (7.2)	314.8 (4.3)	98.42	99.07	-0.28	-0.09	0.997	0.999
Slovakia	2007	5.01 (0.03)	5.32 (0.02)	-215.5 (17.1)	246.9 (4.8)	99.36	99.21	0.26	-0.16	1.002	0.998
	2008	4.99 (0.02)	5.39 (0.02)	-222.5 (14.6)	263.8 (4.9)	99.60	99.27	0.40	-0.13	1.004	0.999
	2009	5.27 (0.02)	5.31 (0.01)	-254.5 (4.6)	290 (5.3)	99.43	99.72	-0.07	0.27	1.000	1.003
	2010	5.24 (0.02)	5.29 (0.01)	-244.2 (5.4)	276.1 (4.5)	99.68	99.68	-0.02	0.17	1.000	1.002
	2011	5.25 (0.02)	5.23 (0.01)	-236.5 (5.2)	272.8 (5)	99.67	99.74	-0.03	0.23	1.000	1.002
	2012	5.19 (0.02)	5.15 (0.01)	-198.8 (5.8)	249.6 (6.7)	99.78	99.88	0.18	0.33	1.002	1.003
	2013	5.18 (0.01)	5.13 (0.01)	-206.2 (5.4)	271.1 (5.7)	99.77	99.88	0.17	0.34	1.002	1.004
	2014	5.21 (0.01)	5.17 (0.01)	-238.5 (4.8)	280.5 (3)	99.70	99.55	0.10	0.15	1.001	1.002
	2015	5.18 (0.02)	5.25 (0.01)	-278.4 (5.3)	287.5 (2.8)	99.70	99.32	0.10	-0.11	1.001	0.999
Slovenia	2010	5.09 (0.03)	4.78 (0.02)	-136.5 (13.1)	138.2 (8)	99.47	99.79	0.37	0.51	1.003	1.005
	2011	5.03 (0.04)	4.8 (0.02)	-99.1 (18)	127.6 (7.7)	99.54	99.76	0.44	0.48	1.005	1.005
	2012	5.11 (0.03)	4.8 (0.02)	-127.1 (13.6)	120 (8.7)	99.64	99.80	0.54	0.51	1.005	1.005
	2013	5.14 (0.03)	4.88 (0.02)	-126.8 (15.2)	149.9 (8.6)	99.30	99.79	0.50	0.60	1.005	1.006
	2014	5.13 (0.07)	5.02 (0.02)	-99.5 (21.5)	189.2 (6.4)	99.60	99.81	0.20	0.58	1.002	1.006
	2015	5.13 (0.07)	5.04 (0.02)	-105.7 (25.9)	187.5 (7.3)	99.51	99.81	0.21	0.54	1.002	1.006

Table 4: Estimated Parameters for Stretched Exponential and Generalized Normal Distributions for Portugal, Romania, Slovakia, and Slovenia.

Country	Year	Parameter d		Parameterλ		Soofi ID		Diff of ID with GN		Likelihood Ratio SE-GN	
		left tail	right tail	left tail	right tail	left tail	right tail	left tail	right tail	left tail	right tail
Spain	2006	4.63 (0.02)	4.79 (0.01)	-64 (5.1)	182.7 (3.5)	99.85	99.85	0.35	0.68	1.004	1.007
	2007	4.51 (0.01)	4.76 (0.01)	-59.3 (3.1)	188.2 (1.9)	99.93	99.82	0.33	0.81	1.003	1.008
	2008	4.72 (0.01)	4.66 (0)	-139.1 (1.6)	158.3 (1.4)	99.82	99.85	0.12	0.71	1.001	1.007
	2009	4.75 (0.01)	4.51 (0)	-131.8 (2)	121.1 (1.3)	99.80	99.88	0.10	0.54	1.001	1.005
	2010	4.72 (0.01)	4.47 (0.01)	-128.7 (1.4)	102.2 (2.2)	99.86	99.87	0.16	0.50	1.002	1.005
	2011	4.78 (0.01)	4.45 (0)	-136.4 (1.5)	101.3 (1.8)	99.87	99.90	0.17	0.44	1.002	1.004
	2012	4.82 (0.01)	4.44 (0.01)	-139 (1.6)	93.7 (1.8)	99.85	99.91	0.15	0.41	1.001	1.004
	2013	4.79 (0)	4.48 (0)	-127.7 (1.6)	94.9 (1.6)	99.90	99.92	0.30	0.40	1.002	1.004
	2014	4.78 (0.01)	4.62 (0)	-123.8 (0.8)	118.2 (1.1)	99.89	99.93	0.29	0.45	1.002	1.004
	2015	4.76 (0.01)	4.69 (0)	-112.3 (1.2)	130.2 (1.4)	99.91	99.92	0.21	0.50	1.002	1.005
Sweden	2007	5.46 (0.01)	5.77 (0.01)	-360.3 (6.9)	280.7 (4)	99.52	98.89	0.22	-0.37	1.002	0.996
	2008	5.45 (0.01)	5.77 (0.01)	-354 (5.2)	280.8 (4.8)	99.58	98.84	0.08	-0.38	1.001	0.996
	2009	5.47 (0.01)	5.74 (0.01)	-315.6 (5.2)	305.9 (4.5)	99.60	99.16	0.20	-0.15	1.002	0.998
	2010	5.38 (0.01)	5.75 (0.01)	-299 (5.1)	288.5 (3.9)	99.74	99.09	0.24	-0.18	1.002	0.998
	2011	5.35 (0.01)	5.76 (0.01)	-311.7 (4.9)	288.4 (4)	99.79	99.12	0.29	-0.18	1.003	0.998
	2012	5.37 (0.01)	5.7 (0.01)	-300.9 (5.1)	276 (4.4)	99.81	99.12	0.21	-0.18	1.002	0.998
	2013	5.34 (0.01)	5.68 (0.01)	-286.2 (4.7)	287.6 (4.2)	99.79	99.27	0.29	-0.09	1.003	0.999
	2014	5.35 (0.01)	5.74 (0.01)	-295.5 (5.6)	300.1 (4.1)	99.81	99.14	0.31	-0.21	1.004	0.998
UK	2007	5.46 (0.02)	5.5 (0.02)	-249 (9)	289.1 (6)	99.55	99.47	0.35	0.41	1.004	1.004
	2008	5.65 (0.01)	5.44 (0.01)	-275.9 (5.9)	289.4 (5.5)	99.39	99.71	-0.01	0.59	1.000	1.006
	2009	5.62 (0.01)	5.32 (0.01)	-255.1 (5.6)	236.3 (11.7)	99.44	99.85	0.14	0.58	1.001	1.006
	2010	4.64 (0.07)	5.23 (0.01)	-25.7 (5.3)	159.6 (5.8)	99.84	99.86	0.24	0.52	1.003	1.005
	2011	4.35 (0.03)	5.26 (0.01)	-14.2 (1)	180.6 (9.7)	99.78	99.85	0.08	0.55	1.001	1.006
	2012	4.33 (0.02)	5.25 (0.01)	-13.9 (0.6)	196.6 (9.2)	99.63	99.68	0.03	0.38	1.000	1.004
	2013	4.37 (0.02)	5.2 (0.01)	-15.3 (0.9)	181.9 (6.1)	99.63	99.81	0.13	0.47	1.001	1.005
	2014	4.37 (0.02)	5.24 (0.01)	-15.8 (0.8)	192.7 (6.6)	99.64	99.84	0.14	0.58	1.001	1.006
	2015	4.4 (0.03)	5.25 (0.01)	-16.6 (1.4)	186.8 (5.6)	99.76	99.82	0.16	0.65	1.002	1.006

Table 5: Estimated Parameters for Stretched Exponential and Generalized Normal Distributions for Spain, Sweden, and the United Kingdom.